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The Design of Mathematics Learning with A Problem Based Learning Scheme to Stimulate Students' HOTS

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ABSTRACT

The need for effective mathematics learning is closely tied to the availability of instructional tools that can foster students' higher-order thinking skills (HOTS). However, such tools remain limited, particularly within vocational education settings. This study aims to develop mathematics learning instruments based on a problem-based learning (PBL) approach that are valid, practical, and effective in enhancing HOTS among vocational high school students. The developed products intended for Grade XI students in the second semester consist of lesson plans, student activity sheets, and learning outcome test instruments. The research employed a 4-D development model, consisting of the stages of define, design, develop, and disseminate. The instruments were tested through three phases: expert review, limited trial, and field trial. Participants included mathematics teachers and students from two different vocational schools. Data were collected using validation sheets, teacher and student assessment instruments, and learning outcome tests. The findings indicate that the developed instruments meet all three criteria: validity, practicality, and effectiveness. Expert validation confirmed that the lesson plans and student activity sheets are valid, the limited trial showed that the tools are practical for classroom use, and the field trial demonstrated that the instruments effectively improved students' HOTS. In conclusion, the developed mathematics learning instruments not only provide an empirically tested and innovative alternative to conventional teaching methods but also contribute significantly to strengthening vocational mathematics education. By promoting students' HOTS, these tools help prepare learners with essential 21st-century competencies.

Keywords: Mathematics Learning; Problem-Based Learning; Higher Order Thinking Skills

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INTRODUCTION

One of the main competencies emphasized in mathematics learning for vocational high school (SMK) students is the ability to think logically, critically, analytically, and creatively, as well as to be careful, responsible, responsive, and resilient in problem-solving (Kemendikbud, 2017). Achieving these competencies requires a transformation in the mathematics learning process. However, mathematics instruction is still largely dominated by conventional approaches focused on memorization and concept application, with little emphasis on developing students' higher order thinking skills (HOTS) (Purwoko et al., 2024; Setiana et al., 2021)

Theoretically, HOTS encompasses critical thinking, creative thinking, and problem-solving skills (Brookhart, 2010). Developing HOTS is crucial so that students can solve complex, non-routine real-life problems (Setiana & Purwoko, 2020). The goal of HOTS-oriented learning is to equip students with reasoning and decision-making skills. Research shows that when students apply HOTS, they can evaluate information, generating new ideas, making predictions, and solving unfamiliar problems (Mohd Tajudin et al., 2018; Pratama & Retnawati, 2018; Retnawati et al., 2018; Riadi, 2016).

Recent international assessments, such as PISA 2022, indicate that Indonesian students' higher order thinking skills (HOTS) in mathematics remain low. In the PISA 2022 assessment, Indonesia achieved an average mathematics score of 366, ranking 68th out of 81 participating countries and economies. This score is significantly below the OECD average of 472. Furthermore, 81.7% of Indonesian students performed below proficiency level 2 in mathematics, indicating a substantial deficiency in essential problem-solving skills. Only 0.04% of students reached proficiency Level 5, and none achieved level 6, which are levels associated with advanced HOTS (Kemendikbudristek, 2023). These results suggest that mathematics instruction in Indonesia still lacks sufficient emphasis on developing students' abilities to tackle non-routine, complex problems that require critical and creative thinking. Local data support these findings. A preliminary survey conducted at a vocational high school in Purworejo revealed that among 32 eleventh-grade students given a HOTS-based question on quadratic equations, almost none were able to answer correctly. Interviews with students indicated they were unfamiliar with such question types, highlighting the lack of exposure to HOTS-oriented problems in the classroom.

The gap between curriculum demands and classroom practices also reflects broader challenges in the Indonesian education system, particularly at the vocational high school (SMK) level. SMK students are expected not only to master academic competencies but also to acquire skills that are directly relevant to the world of work and industry (Kemendikbudristek, 2022). This dual demand makes the development of HOTS even more urgent, as students must be capable of adapting to rapidly changing technological environments, analyzing real-life problems in workplace settings, and making decisions under uncertain conditions. Without adequate mastery of HOTS, SMK graduates may face difficulties in meeting the expectations of the labor market, especially in the era of the Fourth Industrial Revolution (Industry 4.0) and Society 5.0, where creativity, innovation, and critical reasoning are highly valued.

In addition, the learning culture in many Indonesian schools still tends to prioritize examination results rather than cultivating critical and creative thinking processes. National examinations and routine school tests often emphasize procedural knowledge and the ability to apply formulas quickly, rather than problem exploration and reasoning. This assessment orientation indirectly reinforces teachers’ reliance on conventional learning tools and materials, which are not aligned with the development of HOTS. Therefore, there is an urgent need for a paradigm shift, both in terms of learning design and the assessment system, to support the cultivation of HOTS in students.

Developing HOTS requires teachers to design and manage effective learning environments that foster logical thinking, attitudes, and skills. An ideal teacher is one who systematically prepares and implements instructional plans (Sulistyowati et al., 2019). Learning tools are essential components in achieving meaningful learning, enabling instruction that is interactive, engaging, challenging, and supportive of student initiative, creativity, and independence (Islam et al., 2020; Kob et al., 2019; Pratama & Retnawati, 2018). Research by Susanto and Retnawati (2016) shows that well-developed learning tools can enhance student engagement, attitudes, and mastery of mathematical content, particularly in the context of HOTS.

However, preliminary data indicate that many mathematics teachers still do not develop their own learning tools. In a survey of 12 mathematics teachers at a vocational high school in Purworejo Regency, 66.67% reported preparing lesson plans by downloading them from the internet, while only 33.33% created their own. The worksheets used were generally publisher-supplied or downloaded exercises, not designed to foster higher-order problem solving. This situation underscores the need for well-designed learning tools aligned with instructional goals and capable of promoting HOTS development.

One instructional model aligned with both the 2013 Curriculum and the Independent Curriculum, which emphasizes student activity, is Problem-Based Learning (PBL). PBL is defined as a curriculum model built around real-world problems that are often unstructured, open-ended, or ambiguous (So & Kim, 2009). PBL uses real-life problems to promote students' critical and creative thinking in problem solving (Cohn et al., 2017). The PBL stages typically include: (1) presenting the problem, (2) planning the solution, (3) investigating the problem, (4) presenting findings, and (5) analyzing and evaluating results.

PBL has several key characteristics: (1) real problems as the starting point of learning, (2) inquiry-driven instruction, (3) encouragement of student-generated solutions, (4) independent information gathering, (5) use of various sources of knowledge, (6) small group collaboration, (7) teacher as facilitator, and (8) presentation of solutions (Beneroso & Robinson, 2022; Inayah et al., 2021; Liu & Pásztor, 2022; Purwoko, R. Y., Ekawati, D., & Lestari, 2024). This aligns with the theory that HOTS development requires learning environments that include real-world problems, discussion, active student involvement, investigation, and the exchange of ideas (Munawwaroh et al., 2023; Robin Fogarty, 1997)

Key aspects of effective PBL implementation include: (a) use of authentic, unstructured problems, (b) integration of ICT, (c) promotion of independent learning, (d) construction and application of personal understanding, (e) synthesis of knowledge, (f) peer observation, (g) structured study schedules, (h) improvement of communication skills, (i) collaborative problem solving, and (j) use of interactive methods (Susanto & Retnawati, 2016).

Empirical studies support the effectiveness of PBL in improving HOTS. Setyaningsih and Abadi (2018) found that PBL promotes creative thinking, purposeful actions, rational reasoning, effective classroom communication, improved academic achievement, and enhanced mathematical representation and motivation (Farhan et al., 2014a; Seibert, 2021). Other studies have confirmed that PBL fosters student creativity, rational thinking, and problem-solving abilities (Firdaus et al., 2015a; Riadi, 2016; Triasningsih, 2019). Moreover, students engaged in PBL demonstrate stronger abilities in organizing their thoughts and applying knowledge in varied contexts (Raiyn & Tilchin, 2015).

Nevertheless, most of these studies were conducted in general high schools (SMA) or higher education contexts, with limited attention to vocational high schools (SMK), where students face unique challenges and are required to integrate academic knowledge with vocational competencies. In addition, while the literature confirms the potential of PBL to enhance HOTS, relatively few studies have focused on the development of structured learning tools—such as lesson plans, worksheets, or assessment instruments specifically designed to facilitate PBL in mathematics classrooms. Even fewer have contextualized PBL-based problems to vocational fields, which is crucial for ensuring relevance and authenticity in SMK learning environments. This research gap highlights the need for systematic development of PBL-based learning tools that are tailored to the characteristics of vocational students to foster their HOTS effectively.

This study is therefore important as it seeks to address the pressing issue of low HOTS achievement among vocational students in mathematics learning. By developing structured, PBL-based learning tools, this research aims to provide a concrete solution to improve students' critical, creative, and problem-solving skills, while also ensuring that mathematics learning becomes more relevant to the needs of the vocational education context and the demands of the labor market in the era of Industry 4.0 and Society 5.0. These findings underscore the importance of HOTS development in mathematics learning. However, the limited use of instructional tools designed to stimulate HOTS indicates a pressing need for the development of learning tools based on the PBL approach, particularly in vocational high school mathematics contexts.

Despite the potential of PBL, its implementation in Indonesian vocational schools is still relatively limited. Many teachers are not yet fully trained to design PBL-based learning tools that effectively integrate HOTS elements. Additionally, contextualization of problems to match vocational fields (such as engineering, business, or services) is often overlooked, leading to a disconnect between mathematics learning and students' future professional environments. Thus, research on the development of PBL-based learning tools tailored for vocational high schools is highly relevant, both to improve mathematical HOTS and to strengthen the link between mathematics education and industry needs.

The purpose of this study is to develop PBL-based mathematics learning tools that are valid, practical, and effective in enhancing vocational high school students' HOTS. Specifically, the study aims to: (1) design PBL-based mathematics learning tools suited to the characteristics and needs of vocational students, (2) examine their validity through expert judgment and readability testing, (3) assess their practicality through classroom trials, and (4) evaluate their effectiveness in improving students' HOTS.

METHOD

This research is research and development. The product of this development research is a learning instrument with a problem-based learning scheme to stimulate the HOTS of class XI vocational high school students. The instruments developed are learning implementation plans and student activity sheets with learning outcomes test instruments. The development model used in this research is the 4-D model developed by Thiagarajan (Suyitno et al., 2020; Wardani et al., 2019). The study was conducted in June - August 2024 at a vocational high school in Purworejo Regency. The test subjects in this study were experts, teachers, and class XI students from each school. Subjects for the limited and field trials were randomly selected from class XI in each school. In the limited trial, four teachers and 22 students were chosen from each school.

The development procedure in this research refers to four stages: defining, designing, developing, and disseminating. At the definition stage, several analytical activities were carried out as a reference for the initial product design, namely: (1) basic problem analysis on mathematics learning in class and determining alternative problem-solving solutions, (2) student analysis in terms of students' academic characteristics and abilities, (3) material analysis by selecting the competencies developed, and (4) determining the indicators of the selected basic competencies. Furthermore, the product's initial design refers to the results of the analysis carried out. The development of the initial stage is called draft 1.

Three testing stages were carried out at the development stage: expert testing, limited trials, and field trials. An expert test was conducted to determine the validity of the results of the initial design of the learning instrument. Instruments that meet the valid criteria are then carried out with limited field trials. The trial was conducted to determine the practicality of the learning instrument. The results of the revised little test are referred to as draft three, and further field trials are conducted to evaluate the effectiveness of the developed tools. The last stage is dissemination activities by submitting products to schools where research and scientific publications are carried out.

Data collection techniques in this study consisted of tests and non-test. The test technique is carried out by providing an instrument to measure students' HOTS. In contrast, the non-test procedure is carried out through validation sheets, teacher assessment sheets, student assessment sheets, and observation. The instruments used in this study were validation sheets, teacher assessments, student assessments, and learning outcomes test instruments to measure HOTS. The validation sheet measures the instrument's validity using the lesson plan, worksheets, and learning outcomes tests. The teacher and student assessment sheets measured the practicality of learning tools and test learning outcomes. Then the test instrument focused on measuring students' HOTS after learning activities.

Data analysis was conducted to determine the developed learning tools' validity, practicality, and effectiveness. The data from the expert trials were analyzed to determine the validity aspects of the instrument. Learning instruments are said to be valid if the analysis results that have been carried out on average meet the valid category. The analysis of the validity of the learning instrument uses the Aiken validity index with the following formula.

$$V = \frac{\sum s}{n(c-1)}, \quad s = r - I$$

Information:
V = item validity index
r = rater choice category score
I_0 = lowest score of scoring category
c = rater selectable category
n = number of raters

The practicality analysis aims to determine whether the developed learning tools meet the practicality criteria. The learning instrument's practicality was analyzed based on data obtained from the teacher and student assessments in a limited trial. Learning tools are said to be practical if the categories from the results of the analysis of each instrument are minimally practical. The practicality analysis was carried out by converting the data from the trial results on a scale of five, as shown in Table 1.

Table 1. Five Scale Actual Score Conversion

Interval Score	Category
$X > \bar{X}_1 + 1{,}8sb_i$	Very valid
$\bar{X}_1 + 0{,}6sb_i < X < \bar{X}_1 + 1{,}8sb_i$	Valid
$\bar{X}_1 - 0{,}6sb_i < X \leq \bar{X}_1 + 0{,}6sb_i$	Valid enough
$\bar{X}_1 - 1{,}8sb_i < X \leq \bar{X}_1 - 0{,}6sb_i$	Less valid
$X \leq \bar{X}_1 - 1{,}8sb_i$	Not valid

Note:
X = empirical score

$\bar{X}$ = ***ideal everage***

sb = ***ideal standard deviation***

Data analysis was carried out to determine the effectiveness of the learning instrument developed by determining the proportion of students who completed HOTS test results after learning. The steps in data analysis on the effectiveness of learning instruments are (1) tabulating the HOTS scores obtained by students, (2) calculating the number of students who achieve scores above the minimum completeness criteria, (3) concluding the classical percentage of students who achieve the minimum completeness criteria scores. The criteria for the effectiveness of the teaching-learning instrument used in this study is that the product is said to be effective if the percentage of classical student completeness is at least 75%.

RESULT AND DISCUSSION

The initial stage of product evaluation involved an expert review, commonly referred to as a validation test, aimed at assessing the quality of the developed learning instruments in terms of content, structure, and alignment with learning objectives. This validation process was conducted by two experts in mathematics education, who independently reviewed and assessed three components of the learning tools: the lesson plans (RPP), student activity sheets (LKS), and the learning outcomes test instruments. The results of the expert evaluation for the lesson plans, presented in Table 2, indicate that all components met the criteria for validity. Each element within the lesson plans, including learning objectives, activities, assessment methods, and time allocation—was rated in the "valid" category, and the overall average score also reflected a valid classification. Similarly, the analysis of the student activity sheets, as summarized in Table 3, shows that the components assessed such as clarity of instructions, relevance to learning objectives, encouragement of student engagement, and alignment with higher order thinking skills also received ratings within the valid category. These findings suggest that the developed learning tools are well-constructed and appropriate for further implementation in classroom settings.

Table 2. Lesson Plan Validation Results

Aspects Assessed	Score		Aiken Index	Criteria
	1	**2**		
Subject Identity	14	14	0,77	Valid
Formula and Purpose	23	23	0,74	Valid
Material Suitability	24	25	0,68	Valid
Learning model	24	12	0,69	Valid
Learning Activities	26	26	0,7	Valid
Language Accuracy	24	16	0,77	Valid
Learning Resources	14	21	0,73	Valid
Assessment of Learning Outcomes	15	20	0,74	Valid
Application of PBL	22	22	0,77	Valid
Total	186	179	0,72	Valid

Table 3. Validation Results of Student Activity Sheets

Aspects Assessed	Score		Aiken Index	Criteria
	1	2		
Conformity of Content and Material	14	12	0,87	Valid
display settings	23	22	0,82	Valid
Language Component	26	26	0,72	Valid
Components of Serving	14	14	0,67	Valid
Benefits and uses of student activity sheets	22	26	0,67	Valid
Compatibility with PBL	14	14	0,83	Valid
Total	113	114	0,71	Valid

The results of the validator's assessment of the learning outcomes test instrument that measures students' HOTS are valid criteria. The data from the assessment results are based on two aspects, namely the suitability of indicators and editorial questions, as shown in the following table.

Table 4. Validation of Learning Outcomes Test Instruments

Aspects Assessed	Score	Score Validator		Criteria
		1	2	
Indicator suitability	46	46	46	Valid
Question editor	46	46	46	Valid
Total	92	92	92	Valid

1. Limited Trial Results Data

The product validated and revised based on suggestions from the validator in draft 2 is then carried out with limited trials to determine the instrument's practicality. The trial was conducted in two schools that were the subject of the study. The results of the limited trial consist of two, namely the results of teacher assessments and student assessments. In the teacher's assessment, two math teachers of class XI were selected from each school. The results of the teacher's assessment of learning tools are shown in Table 5.

Table 5. Teacher's assessment of learning instruments

Evaluator	**Total Score**		
	Lesson plan	**Student activity sheet**	**Study result test**
Teacher 1	35	34	31
Teacher 2	35	40	32
Teacher 3	34	38	30
Teacher 4	35	36	30

Everage	34,75	37	30,75
Criteria	Very Practical	Practical	Practical

The assessment results show that, on average, the learning instruments meet the practical criteria so that it can be concluded that the instruments are feasible to use. Furthermore, the results of student assessments of learning tools were analyzed from the assessments of six students from each school. The data on student assessment results (readability test) on learning tools in the form of student worksheets and learning outcomes test instruments are shown in Table 6.

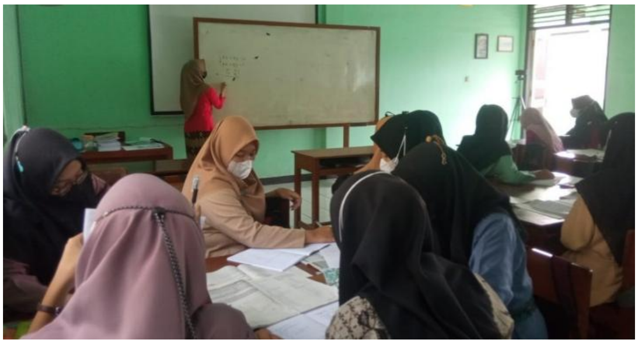
Table 6. Student assessment of learning instruments

Aspects Assessed	Max Score	Everege Score	Category
Student worksheets	45	39,25	Practical
Learning outcomes test	36	23,75	Practical

2. Field Trial Result Data

Learning instrument products tested by experts and limited trials are from now on referred to as draft 3, and field trials are carried out. The trial class chosen was in class XI at two public vocational schools in Purworejo Regency with two math teachers. The following is the implementation of field trials in 2 different vocational schools.

Figure 1. Implementation of Learning Using the Developed Instruments



Learning was carried out in six meetings, and at the end of the meeting, a test was given, which consisted of five multiple-choice questions and five essay questions that measured students' HOTS. The data from field trials are shown in table 7 below:

Table 7. Field trial results data

Schools	Students	Average	Percentage (%)
School 1	28	78,00	82,00
School 2	30	80,00	80,00

Based on the table above, the percentage of students' learning completeness classically achieves the Minimum Mastery Criteria, which is more than 75%. In addition, the average score of the two test classes has reached the Minimum Adequacy Criteria. The percentage of completeness in school 1 is 82%, with an average score of 78.00, and the percentage of completeness in school 2 is 80.00%, with an average score of 80.00. This shows that the learning tools developed have met the criteria of effectiveness.

3. Product Revision

At each trial stage, revisions were made to obtain the feasibility of the product for use. In expert trials, the product is revised based on the suggestions given by the validator. Improvements to the RPP include improving indicators, core activities, opening activities, apperceptions, information on when to divide groups, and including conclusions in closing activities. On student activity sheets (LKS), in general, suggestions for improvement from the validator, such as writing time allocations, presenting problems, and presenting pictures. Furthermore, on the learning outcomes test instrument, improvements are carried out per the validator's suggestions, including the order of answers and units of account in the answer choices. Revisions in improving learning tools are based on the results and suggestions from limited trials in general only on writing command sentences in worksheets, question sentences in test instruments, and image layouts in worksheets. Revisions were made based on suggestions and input from the teacher. At the field trial stage, revisions were also based on the teacher's findings and suggestions when using learning instruments.

4. Final Product Review

Mathematics learning tools with PBL schemes to stimulate HOTS were developed, referring to the findings on the background and the analysis results at the definition stage. Based on the analysis results, media selection, format selection, and initial product design were carried out. The initial product design is then carried out in the trial stages, namely expert, limited, and field trials. At each trial stage, revisions or improvements are made so that the developed product meets eligibility. The feasibility of using the instrument is assessed based on three aspects, namely: (a) validity, (b) practicality, and (c) effectiveness (Purwoko & Santosa, 2020).

As a tangible output, the developed product consists of lesson plans (RPP), student activity sheets (LAS), and learning outcome test instruments, all of which are systematically designed according to the principles of Problem-Based Learning (PBL) and oriented toward the development of HOTS. The lesson plans are arranged to follow the stages of PBL, starting with the presentation of contextual, unstructured problems, continued with group discussions, guided inquiry, solution presentations, and reflection. For example, in the topic of *Linear Programming*, students are confronted with contextual problems related to production optimization in vocational settings (e.g., a workshop producing chairs and tables with limited resources). The student activity sheets are designed to guide students in translating real-world problems into mathematical models, identifying variables, formulating constraints, graphing solution areas, and interpreting the results in practical contexts. Meanwhile, the learning outcome test instruments are intended to measure students' HOTS at the levels of analysis, evaluation, and creation, such as evaluating the feasibility of certain production strategies or designing optimal solutions based on mathematical reasoning.

Learning tools: lesson plans, worksheets, and learning outcomes test instruments developed meet valid criteria based on content validity by two validators. This shows that the components of the designed learning instrument are in accordance with the indicators set on the learning instrument's validity instrument. The learning tools developed also have strong theoretical relationships and internal consistency between the components being developed (Susanto & Retnawati, 2016). The following is a summary of the developed products, presented in Figure 2.

Figure 2. Summary of Learning Design Scenarios

Lesson Plan (RPP)

Mata Pelajaran: Matematika
Kelas/Semester: XI / 2
Materi: Program Linear (Masalah Optimasi)
Pendekatan/Skema: Problem-Based Learning (PBL)
Sintaks PBL:

1. **Orientasi masalah**: Guru menyajikan permasalahan kontekstual.
   "Sebuah bengkel memproduksi kursi dan meja. Kursi membutuhkan 2 jam kerja dan 1 papan kayu, sedangkan meja membutuhkan 3 jam kerja dan 2 papan kayu. Bengkel hanya memiliki 60 jam kerja dan 40 papan kayu. Keuntungan kursi Rp50.000, meja Rp70.000. Tentukan kombinasi produk yang memberi keuntungan maksimum!"
2. **Mengorganisasikan siswa**: Siswa dibagi kelompok, mendiskusikan pemahaman masalah.
3. **Membimbing penyelidikan**: Guru membantu siswa mengubah masalah ke model matematika (pertidaksamaan linear, fungsi objektif).
4. **Mengembangkan dan menyajikan hasil karya**: Kelompok mempresentasikan grafik daerah penyelesaian & hasil optimasi.
5. **Menganalisis dan mengevaluasi**: Guru dan siswa refleksi strategi penyelesaian, menghubungkan dengan konteks nyata.

Student Activity Sheet (LKS) – Contoh Aktivitas

Judul: Optimasi Produksi dengan Program Linear
Petunjuk Siswa:

1. Pahami masalah kontekstual yang diberikan.
2. Identifikasi variabel keputusan.
3. Susun model pertidaksamaan linear.
4. Gambarkan grafik daerah solusi.
5. Tentukan titik pojok, hitung nilai fungsi objektif, lalu tentukan solusi optimal.

Pertanyaan Diskusi:

- Apa keterbatasan sumber daya pada masalah tersebut?
- Bagaimana cara menentukan kombinasi terbaik?
- Apakah hasil model matematika masuk akal jika diterapkan di dunia nyata?

Learning Outcome Test Instrument (Contoh Soal HOTS)

Soal 1 (C4 - Menganalisis)
kertas, brosur memerlukan 2 jam kerja dan 2 rim kertas. Tersedia 80 jam kerja dan 60 rim kertas. Tentukan model matematika program linear untuk masalah ini.
Soal 2 (C5 - Mengevaluasi)
Analisis apakah mungkin memproduksi 10 poster dan 20 brosur dengan sumber daya yang ada. Jelaskan alasanmu.
Soal 3 (C6 - Mencipta)
Jika keuntungan per poster Rp15.000 dan per brosur Rp10.000, tentukan strategi produksi optimal agar keuntungan

The practicality of learning tools is based on the readability aspect of the teacher's and student's assessments. Based on the data analysis, it shows that the learning tools developed meet the practical category. This is shown by the average score of student assessments in the practical category, while the average score of the teacher's evaluation of learning tools is in the very practical category. The product's practicality is very important because it will be easier for teachers and students to learn using the instruments that have been developed (Gularso et al., 2023; Suyitno et al., 2020).

Learning instrument products, namely lesson plans and worksheets, meet the criteria for being effective. This means that there is consistency between the learning tools set and implemented as well as the learning tools set and the goals to be achieved. The effectiveness criteria were met based on the analysis of learning achievement test data that measured students' HOTS. Analysis of students' HOTS test results showed that the average percentage of completeness exceeded the established criteria, namely 75%. The percentage of completeness in school 1 is 82%, with an average score of 78, while the percentage of completeness of students in school 2 is 80%, with an average score of 80. Thus, the product being developed has positive potential for the development of student HOTS; this is also following research which states that student activity in mathematics must be oriented towards developing student HOTS because this is the final achievement of real learning (Kwangmuang et al., 2021; Setiana & Purwoko, 2020).

Based on the review of the three aspects above, it can be concluded that the mathematics learning tool with the PBL scheme to stimulate HOTS meets the valid, practical, and effective criteria. This shows that the developed instrument is feasible to use in learning mathematics. This follows the opinion (Nieveen & Folmer, 2013), which states that the quality aspects of materials for consideration of learning instruments must consider three aspects: validity, practicality, and effectiveness.

DISCUSSION

The findings of this study demonstrate that the development of PBL-based mathematics learning tools for vocational high schools has met the criteria of validity and practicality, thereby providing a strong foundation for their application in classroom practice. The learning tools including lesson plans, student activity sheets, and assessment instruments were systematically designed and evaluated, showing that they are both feasible and relevant to the learning needs of vocational school students. This result aligns with previous studies that have emphasized the effectiveness of problem-based learning (PBL) in fostering higher-order thinking skills (HOTS). However, the present study advances the discourse by explicitly linking the design of learning tools to the vocational education context, thereby addressing specific gaps in the literature.

Empirical studies have consistently reported the effectiveness of PBL in improving HOTS. Setyaningsih and Abadi (2018) found that PBL promotes creative thinking, purposeful actions, rational reasoning, effective classroom communication, improved academic achievement, and enhanced mathematical representation and motivation (Farhan et al., 2014b; Seibert, 2021). Other studies have confirmed that PBL fosters student creativity, rational thinking, and problem-solving (Firdaus et al., 2015a; Riadi, 2016; Triasningsih, 2019). Moreover, students engaged in PBL demonstrate stronger abilities in organizing their thoughts and applying knowledge in varied contexts (Raiyn & Tilchin, 2015). Yet, while the effectiveness of PBL has been widely acknowledged, fewer studies have systematically integrated HOTS into the design of learning tools validated specifically for vocational high schools. This study fills that gap by not only confirming the practicality of PBL-based instruction but also providing a structured and validated framework for its implementation in vocational education.

The contribution of this study can be distinguished from earlier research in several important ways. First, while previous investigations (Farhan et al., 2014a; Setyaningsih & Abadi, 2018) highlighted the role of PBL in enhancing general problem-solving abilities, they often did not provide a systematic validation process for the instructional instruments. The present study conducted both expert validation and field practicality testing, ensuring that the tools were not only theoretically grounded but also functional in authentic classroom contexts (Restika Rahmadani & Sari Herlina, 2020). Second, whereas studies such as (Firdaus et al., 2015b; Raiyn & Tilchin, 2015) focused primarily on the cognitive outcomes of PBL, the current study emphasizes the alignment of instructional tools with the explicit development of HOTS, making the design process more intentional and outcomes driven. Third, prior studies (Seibert, 2021; Triasningsih, 2019) largely examined traditional secondary schools, while this study targeted vocational high schools, a context often overlooked despite its critical role in preparing students for both higher education and the workforce. Fourth, compared to research that measured effectiveness solely based on student outcomes (Purwoko et al., 2023), this study combines validity, practicality, and future potential for effectiveness, providing a more holistic approach. Fifth, while (Brookhart, 2010) and others highlighted the importance of aligning assessment with HOTS, this study operationalizes such alignment within a comprehensive instructional package, ensuring coherence across objectives, activities, and evaluations. Sixth, unlike earlier research conducted in Western contexts (Seibert, 2021), this study provides evidence from the Indonesian vocational education system, contributing localized insights that enrich the global literature on PBL. Seventh, compared with studies focusing only on teacher perspectives (Riadi, 2016), this study incorporates both teacher and student evaluations of practicality, thereby offering a more balanced assessment. Finally, the study situates its contributions within the broader policy context of Indonesia’s Merdeka Belajar initiative, something rarely addressed in prior PBL research.

The implications of these findings are both theoretical and practical. Theoretically, this study reinforces the argument that PBL, when systematically designed and supported by validated instructional tools, can effectively promote HOTS among vocational high school students. This provides evidence for expanding the theoretical framework of PBL beyond general education into vocational education, which has unique curricular demands. Practically, the validated tools developed in this study can serve as exemplars for teachers seeking to implement PBL in mathematics classes. The fact that both teachers and students assessed the tools as practical highlights their adaptability to real teaching environments. Moreover, the structured integration of lesson plans, student activity sheets, and assessment instruments provides a coherent package that ensures alignment between learning objectives, activities, and assessments (Brookhart, 2010; Bwalya & Rutegwa, 2023; Purwoko, 2017). For policy makers, the findings support the integration of systematically designed instructional tools into the broader framework of Indonesia's curriculum reform, particularly in supporting the cultivation of 21st-century skills and competencies aligned with the vision of Indonesia Emas 2045.

Another implication concerns teacher professional development. The findings suggest that the capacity of teachers to design and implement validated PBL-based learning tools is crucial for sustaining effective classroom practice. Therefore, professional development programs should include training in instructional design, particularly in aligning learning objectives with HOTS-based tasks. In addition, collaboration between teachers and curriculum developers could ensure that learning tools remain contextually relevant and adaptable to diverse classroom settings. On a broader scale, these results contribute to global educational discourse by illustrating how PBL can be systematically operationalized in developing countries, thereby expanding the international relevance of PBL research.

Despite its contributions, this study has several limitations that must be acknowledged. First, the validation process was limited to two expert validators in mathematics education, which may constrain the comprehensiveness of the content and pedagogical evaluation. Engaging a larger and more diverse group of validators could enhance the robustness of the findings (Aiken, 1985; Imelda et al., 2019; Puji Astuti et al., 2021; Purwaningrum et al., 2023; Purwoko et al., 2024). Second, the practicality testing was conducted on a small scale, involving only two vocational schools and a limited number of teachers and students. This restricts the generalizability of the findings, as vocational schools in different regions may face varying challenges, such as disparities in resources, student preparedness, and institutional culture. Third, the study focused on short-term evaluations of validity and practicality without conducting longitudinal assessments of the tools' effectiveness. As a result, the long-term impact of the tools on sustained HOTS development remains to be empirically verified. Finally, the study employed a predominantly quantitative approach in assessing validity and practicality, which may have overlooked richer qualitative insights into classroom dynamics, student motivation, and teacher experiences.

These limitations open avenues for further research. Future studies should conduct large-scale trials across diverse educational settings to examine the broader applicability of the developed learning tools. Longitudinal studies would be particularly valuable to assess the sustainability of the tools' effectiveness in fostering HOTS over time. Additionally, mixed-method research designs could provide more comprehensive insights by triangulating quantitative outcomes with qualitative data on classroom processes. Another promising direction is the integration of technology into the learning tools, such as digital platforms or AI-assisted learning systems, which could enhance engagement and provide adaptive feedback (Nuryadi et al., 2023; Raiyn & Tilchin, 2015; Wibowo et al., 2023). Considering the increasing role of digitalization in education, exploring hybrid models that combine PBL with technology could be highly beneficial. Moreover, comparative studies across different cultural and educational contexts could further enrich the global understanding of how PBL functions in diverse settings, thereby contributing to a more nuanced theory of PBL in mathematics education.

In conclusion, the present study contributes to the growing body of literature on PBL by systematically developing and validating mathematics learning tools designed to foster HOTS in vocational high schools. Distinct from prior research, this study emphasizes the integration of validity, practicality, and contextual relevance, providing both theoretical and practical contributions to mathematics education. While limitations remain, the findings highlight the potential of PBL-based instructional tools to address persistent challenges in vocational education, particularly the need to cultivate higher-order cognitive skills essential for the 21st century. Future research should expand on these findings through larger-scale, longitudinal, and technology-enhanced studies, thereby ensuring the continuous refinement and global applicability of PBL in mathematics education.

CONCLUSION

This study aimed to develop mathematics learning instruments based on Problem-Based Learning (PBL) to enhance the Higher-Order Thinking Skills (HOTS) of vocational high school students. The findings indicate that the developed instruments possess three main characteristics: (1) they are oriented toward authentic, ill-structured problems relevant to real-life and workplace contexts, (2) they are systematically designed according to the stages of PBL and scientific inquiry, and (3) they are specifically intended to stimulate HOTS, including critical thinking, problem-solving, reasoning, and creativity. The developed product meets the validity criteria, as evidenced by expert evaluations categorizing all components as "valid." In terms of practicality, the instruments received "very practical" ratings from teachers and "practical" ratings from students, demonstrating their ease of use and feasibility in classroom implementation. Furthermore, the product was shown to meet the effectiveness criteria, as reflected in the percentage of classical learning mastery, thereby indicating its potential to improve both student achievement and cognitive engagement. This research contributes to both theory and practice. Theoretically, it addresses the research gap in applying PBL within mathematics learning at vocational schools. Practically, it provides a feasible, evidence-based alternative to conventional teaching methods, while also empowering teachers to foster 21st-century competencies. Nevertheless, this study has limitations, including the restricted scope of implementation and the specific instruments used to measure HOTS. Future research is recommended to broaden its application and integrate digital technologies or artificial intelligence to further optimize the implementation of PBL.

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AUTHOR CONTRIBUTION STATEMENT

RYP carried out research design, data collection and manuscript design. JPP, AG, and SH carried out analysis and writing of the final manuscript.

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