Critical Thinking Skills of Students in Solving Mathematical Problem

Ardi Dwi Susandi

Nahdlatul Ulama University Cirebon, Indonesia
Correspondence: dwisusandiardi@gmail.com

Abstract

Mathematical critical thinking skills are one of the essential skills in the 21st century. However, in reality, many students still do not have good mathematical critical thinking skills. This is because the teacher has not been able to process students' critical thinking in mathematics. Most teachers only teach mathematical concepts without seeing how the students' critical thinking processes in solving mathematics problems are given. Therefore, it is necessary to know how the critical thinking process of students so that teachers know what strategies should be applied in the classroom. This study aims to (1) describe the critical thinking skills of students in Grade 8 in solving a system of linear equations in two variables (SLETV) and (2) identify the components of critical thinking in students. It is a study with a qualitative descriptive approach. Three indicators are used as the basis, namely analyzing, evaluating, and drawing a conclusion. As many as 32 subjects were involved (14 males and 18 females). The research instrument was a test with three problems containing three indicators. Data were collected through test and interview methods, and triangulation was done to compare the outcomes between tests and interviews. The data were analyzed through data reduction, data display, and conclusion drawing. The findings indicate that: (1) critical thinking skills of students are classified 'low'; (2) students' ability in concluding, as one indicator of critical thinking skills, is lower than other indicators.

INTRODUCTION

Critical thinking is one of the 21st Century skills that today's students need to master for life [1]–[6]. It has been extensively recognized as one of the essential life skills [7]–[9]. Individuals having such skills relatively own many benefits. They can analyze specific situations and decide [10], [11]. They can think critically and notice and solve problems efficiently [12], [13]. Critical thinking enables students to process information logically, assisting them in learning independently [14]. Otherwise, students who graduate without critical thinking skills will have issues competing in work and society [15].

Considering the role of critical thinking is essential to include such skills in the current education system [16]. These skills have been the most expected learning outcome [17], [18]. These skills facilitate students to exercise their concentration, argue, make comprehensive analyses for concluding and solving problems [12]. One of the subjects to enhance critical thinking skills in mathematics [19] is developed through cognitive processes in solving problems and affecting students' attitudes towards the subject [20]. Despite its role, most students around the world have low critical thinking skills.

Similar to students' critical thinking skills in various countries, the Indonesian students also indicate low skills. This low-performance level is identified by the results of the Programme for International Student Assessment (PISA), which shows that Indonesia ranked 69th out of 75...
countries in 2015 [21]. Likewise, the Trends in International Mathematics and Science Study (TIMSS) published the ranks of Indonesian students in scientific literacy in 1999, 2003, 2007, and 2011, namely 32nd out of 38, 37th of 46, 35th of 49, and 40 of 42 countries, respectively.

Many factors become the causes of the low critical thinking skills of Indonesian students. Based on preliminary studies conducted at several schools, it can be summarized that students only learn to memorize concepts and theories while posed to problems that do not stimulate their critical thinking skills. They are accustomed to a teacher-centered approach instead of exploring knowledge individually. In addition, an analysis of the items in the National Examination (UN) reveals the contents have not sufficiently measured students' critical thinking skills. As a parameter, multiple-choice questions in the UN are incapable of measuring the thinking processes. Many questions cannot be categorized as problems that aim to measure critical thinking skills since they generally emphasize the application of formulas or theorems and memorization skills. The Indonesian government attempts to incorporate critical thinking skills in all subjects (Kemendikbud, 2013). Furthermore, it is stipulated in Education Minister Regulation Number (Per mendikbud) No. 20 of 2016 concerning the six thinking skills that students must achieve in the competency standards of elementary and secondary education graduates, one of which is critical thinking.

Several scholars have explained the definition of critical thinking skills. Critical thinking is logical, reflective thinking skill that focuses on determining what should be done [7], [22], [23]. Chaffe (2012) states that critical thinking is the mental process to clarify one's understanding in making an accurate decision [23]. Critical thinking is the logic and reflective thinking that involves analyzing, evaluating, and drawing conclusions to make correct decisions.

Based on the definition above, the abilities to analyze, evaluate arguments, and conclude are essential for possessing good critical thinking skills [20], [22], [24]–[27]. Problems that require students to think critically are those related to analyzing, evaluating, and making conclusions [28]. Therefore, the critical thinking indicators in the present study include analyzing, evaluating, and drawing conclusions.

Several studies have discussed critical thinking skills in real life. Carry out a study on mathematics teacher candidates in elementary school, disclosing their moderate skills level. The obstacles faced by most teacher candidates include the efforts to improve students' critical thinking skills. Kumar and James explain that male students relatively have higher critical thinking skills than female students [29]. Similarly, Moeti et al. claim that students have an inadequate description of critical thinking problems due to the lack of practicing during learning [30]. Fazriyah et al. report the significant differences in science learning outcomes of the 5th grade of elementary school students in critical thinking skills [31]. Schreglmann and Ozturk explicate the student's negative perceptions toward such skills [32]. Meanwhile, Kusaeri and Aditomo reveal that only 60% of teachers are convinced about applying critical thinking assessments [33].

The findings reported from previous studies have not described the critical thinking skills and the sub-skills of students' critical thinking. Therefore, this study aims to: (1) describe the critical thinking skills of students in Grade 8 in solving the system of linear equation in two variables (SLETV), and (2) identify the components of critical thinking in students, namely: analyzing, evaluating and drawing a conclusion. Initial identification is performed to investigate students' critical thinking skills; hence the result will be helpful for teachers in developing instructional learning models to improve these skills.
METHODS
Type of Research
The present study is a qualitative descriptive study. A qualitative approach is used to understand the subjects’ context [34]. The critical thinking skills of 32 students in Grade 8 are investigated and explained.

Research Subjects
Junior high school students (Grade 8) were involved as the research subjects. The school is an A-accredited state junior high school, a favorite school that has generated students with accomplishments at a national scale. The class with the most outstanding performance was selected. There were 32 students (14 males and 18 females) were involved.

Research Instrument
Two types of instruments were employed, namely the primary instrument and secondary instruments. The researchers’ primary instrument was the planners, data collectors, analysts, interpreters, and reporters. The secondary instruments tested students’ critical thinking skills in solving linear equations in two variables (SLETV), assessment rubric and interview guidelines. The test consists of three problems to measure students’ abilities to analyze, evaluate, and draw conclusions. In addition to the test outcome, the interview is used to validate the data. The test is presented in Table 1.

<table>
<thead>
<tr>
<th>No</th>
<th>Component of Critical Thinking</th>
<th>Indicator</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Analyze</td>
<td>Identify all the information to formulate the problem and the correct answer</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Evaluate</td>
<td>Assess answers correctly and give reasons correctly</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Draw conclusion</td>
<td>Make the proper conclusion and give the proper reason</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total Item</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

Furthermore, for the assessment rubric, the scores are determined by the indicators of critical thinking skills set previously. By using this rubric, the critical thinking skills level of the subject is identified. In addition, an interview instrument is designed to clarify and verify the subject’s test worksheet and the possible reasons for the response. It is designed as an unstructured interview. The scoring rubric for each critical thinking indicator can be seen in Table 2.

<table>
<thead>
<tr>
<th>Measured Aspect</th>
<th>Description</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis</td>
<td>Not answering questions</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Identify some of the information to formulate the problem and the wrong answer</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Identify all the information to formulate the problem and the wrong answer</td>
<td>2</td>
</tr>
</tbody>
</table>
Procedure

As many as 32 students of Grade 8 in a state junior high school-aged 13-14 were involved in the present study. They were required to complete a mathematics test to score their critical thinking skills. The duration was 30 min. Subsequently, the collected worksheets were codified (numbers 1 to 32). The answers were scored based on the assessment rubric. In addition to the written test, interviews were conducted with 15-25 min for each subject.

Data Analysis

Data analysis is done in three stages. First, data obtained from the test and interviews are reduced by selecting the significant while eliminating the insignificant data. Second, data from the written test are analyzed by calculating the percentage. In addition, some points from the test and interview are explicated descriptively and displayed in three parts, namely: (a) critical thinking skills of students in analyzing, (b) critical thinking skills of students in evaluating, and (c) critical thinking skills of students in concluding. The criteria for the mean score of students’ critical thinking skills are presented in Table 3.

<table>
<thead>
<tr>
<th>Average Score</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>75% &lt; P ≤ 100%</td>
<td>Good</td>
</tr>
<tr>
<td>60% &lt; P ≤ 75%</td>
<td>Fair</td>
</tr>
<tr>
<td>P ≤ 60%</td>
<td>Low</td>
</tr>
</tbody>
</table>

RESULTS

Based on the analysis of the results of tests and interviews, students' critical thinking skills for each indicator are obtained.
The Ability to Analyze

Based on the analysis, only eight students (25%) can analyze accurately, while 24 students (75%) cannot do it appropriately. The worksheet and the interview with a student showing low critical thinking skills in analyzing are clarified below.

Figure. 1. Answer Subject S7

Interview with Subject 7
R: Are you sure with your answer? (Pointing out the worksheet)
S7: Yes, I am.
R: Take a look at your answer.
S7: (Re-examining the final answer)
R: So?
S7: I think my answer is correct.
R: Well. Check the problem. What is the question? (Showing the question).
S7: How much is the total cost that Nando should pay as a member of BPJS Class 2 for the late payment of four months?
R: Okay. Look again at your answer.
S7: (S7 re-examine the worksheet)
R: What is the premise?
S7: x for member of BPJS Class 2 and y for Class 3.
R: Recheck your answer.
S7: (Checking the worksheet).
R: Do you realize whether your answer is by the given problem or not?
S7: Yes, I do. I made a mistake. I solved the problem for the BPJS Class 3, while the question is the late payment for the member of BPJS Class 2.
R: Well. Recheck your answer.
S7: (Checking the worksheet).
R: Is there any incorrect calculation in your answer?
S7: The subtraction of \( y \) should be \( 40y \) instead of \( 20y \).
R: Anything else?
S7: The result of \( 4.590.000 - 663.000 \) should be \( -2.040.000 \) instead of \( -1.040.000 \).
R: Anything else?
S7: In calculating the cost that Nando should pay, I did not multiply x to 4.
R: Anything else?
S7: No, Sir.
R: Okay.

The worksheet and interview above show that the subject has not been able to analyze a problem. It is verified by the inaccuracy in solving the given problem based on the information. Nevertheless, the subject realizes the answer is incorrect. It takes place the subject's inadequate understanding of the purpose of the matter. Moreover, the subject also makes a calculation error in subtraction. In addition, there is also a mistake in determining the cost that Nando should pay for the late payment of four months. The subject only multiplies the number of months (four months) to the acceptable amount without including the premium cost. The subject has a misconception as well as calculation error.

The Ability to Evaluate

Based on the analysis of the students' ability to evaluate, only six students (18.75%) can evaluate adequately, while 26 students (81.25%) are not. The worksheet and the interview with students showing low critical thinking skills in evaluating are clarified below.

Figure 2. Answer Subject S15

Interview with Subject 15
R: Do you solve the problem correctly? (Pointing out the answer).
S15: I do, Sir.
R: Retake a look.
S15: (Re-examining the worksheet).
R: So?
S15: I do believe the answer is correct.
R: Well. Read the problem. What is the question? (Pointing out the problem).
S15: "Does Andi give a correct solution for the problem? Explain your reason!"
R: Let us take a look at your answer.
S15: (Re-examine the answer).
R: Did you answer the question correctly?
S15: (Thinking).
R: So?
S15: No. It did not, Sir.
R: So, how did you solve the problem?
S15: I determined the values of \( x \) and \( y \) then inputted them into the equation.
R: Why did you take the measure?
S15: I did not understand the given problem, Sir.
R: Well, have you ever solved this kind of problem previously?
S15: Not yet. I just have it.
R: Well then.

The worksheet and interview above show that the subject has not mastered the ability to evaluate. The subject is incapable of processing the information in the given problem; hence the solution is provided based on the subject's understanding. In addition, the subject fails to recognize the mistakes in the solution. The interview reveals that the subject has not previously posed such problems and does not understand how to solve SLETU using the substitution method. The subject imprecisely takes the steps in using the substitution method.

**The Ability of Drawing Conclusion**

Based on the analysis of students' ability to conclude, only three students (9.375%) can conclude accurately, while 29 students (90.625%) are not. The worksheet and the interview with students showing low critical thinking skills in evaluating are clarified below.

![Figure 3. Answer Subject S15](image)

**Interview with Subject 25**

R: Are you sure with your answer? (Showing the worksheet to the subject).
S25: Yes, I am.
R: Retake a look.
S25: (Re-examining the worksheet).
R: So?
S25: I have provided a correct answer.
R: If you said so. Read the problem. What is the question? (Pointing out the given problem).
S25: "What is the conclusion from the answer? Explain your argument to support the conclusion!"
R: Let us take a look at your answer.
S25: (Re-examine the worksheet).
R: Does your answer solve the question in the given problem?
S25: (Thinking)
R: So?
S25: Yes, it does.
R: Recheck your answer!
S25: (Re-examine the answer).
R: Can you define the meaning of 0 ≠ 0?
S25: I am sorry. It should be 0 = 0.
R: So, what is the meaning?
S25: There are many solutions.
R: Okay, so what is the conclusion?
S25: There are many solutions to the problem.
R: What is your argument?
S25: Because 0 = 0.
R: Well then.

The answer and interview provided by the subject reveal the inability of the subject to conclude accurately. The conclusion is neither related to the given problem nor supported by a valid reason. However, the subject can solve SLETv using the elimination method. It is indicated by the ability of the subject to carry out the correct steps of the method. The subject also understands the meaning of 0 = 0, informing it shows the infinitely many solutions for SLETv, despite the subject's inability to provide an example of the solution. The recapitulation of the critical thinking skills of students in Grade 8 is presented in Table 4.

<table>
<thead>
<tr>
<th>No.</th>
<th>Critical Thinking Sub-Skills</th>
<th>Percentage of the students with good critical thinking skills</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Analyze</td>
<td>25%</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Evaluate</td>
<td>18.75%</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>Draw conclusion</td>
<td>9.375%</td>
<td>Low</td>
</tr>
</tbody>
</table>

Based on the three indicators of critical thinking skills shown in Table 4, students' skills can be classified as low. Particularly the ability to conclude, which obtains the lowest percentage. It indicates that most students have not been able to solve the problems related to critical mathematical thinking.

**DISCUSSION**

Based on the results of data analysis, it can be stated that the critical thinking skills of students in Grade 8 are relatively low. It is demonstrated by the low level of all three indicators who conclude students' low critical thinking skills [35]–[37]. It is mainly caused by the common conception of critical thinking skills among students. In addition, teachers hardly practice the learning activity that involves critical thinking skills in class [38]. It can be improved by directing students to deal with non-routine problem solving that requires critical thinking [39]. This finding is expected to
encourage mathematics teachers to design instructional approaches that improve students' critical thinking skills, especially in mathematics.

The low mathematical critical thinking skills of students also imply the level of critical thinking sub-skills. It turns out that students have not been able to analyze accurately. It is indicated by students' incapability to analyze, in which the students should be able to identify information, formulate the problem, and solve it correctly. Even though they can construct a system of linear equations and write questions in a problem correctly, they do not provide answers according to the question. It occurs because they do not recognize the goal of the given problem. In addition, students' lack of knowledge leads them to misinterpret the goal of a problem. However, since students understand the concept of solving completing SLETV with the elimination method, they can complete the steps of the method correctly. However, students tend to make calculation errors in subtraction while doing the elimination method. It is common for students to be careless in calculations. Most students have misconceptions about mathematical calculations since they are less careful in calculations [40], [41]. In the present study, the subject determines the BPJS cost that Nando should pay for the late payment of four months. The student incorrectly solves the problem by only multiplying the amount of the fine to four without including the premium cost. It indicates students have difficulties in understanding a concept and further making calculation errors.

The analysis also reveals that most students have not been able to evaluate correctly. The indicators, which include evaluating and arguing the answer based on the evaluation, have not been met. It is implied from students' incapability in identifying the errors in the given problem, leading to the failure to provide an argument. It occurs because they do not understand the goal of the problem, and consequently, they solve the problem randomly. In addition, problems of SLETV are rarely discussed by teachers. The research subjects even have not been posed with such problems previously. It is exacerbated by the fact that students do not understand how to solve SLETV using the substitution method, as reflected by the errors made during problem-solving. It occurs because they do not have a comprehensive understanding of mathematical concepts in solving mathematical problems.

This study reveals that most students have not concluded accurately in the context of the ability to conclude. The indicators, which include the ability to conclude and argue based on the acquired information, have not been fulfilled. The present study shows that the students fail to conclude the problem. Consequently, they are not able to argue and defend the conclusion. Nevertheless, they can solve SLETV using the elimination method as represented by the worksheet and interview. They understand the meaning of $0 = 0$, which shows the SLETV has infinite solutions, but they cannot provide an example. They also mention that SLETV with many solutions represented by a graph with two imposed lines. It indicates that students employ many ways to solve mathematical problems. Therefore, noticing different forms of representation will help students solve problems with diverse alternatives [42].

Based on the explanation above, it can be claimed that students' low critical thinking skills are linked to a misconception, procedural errors, and calculation errors. Such errors allegedly occur due to the method of mathematics learning, which prioritizes memorizing instead of understanding [43]. Several difficulties are undergone by the student, i.e., to absorb the information correctly, the lack of experience in working on mathematical problems, understanding the given material comprehensively, and weak prerequisite skills [44]. Therefore, mathematics teachers must explore the aspects of errors in the class [45]. It is an effective effort to prevent students from making...
similar errors and enhance their critical thinking skills. Several techniques are helpful to improve these skills, particularly for secondary education and higher education levels, namely: learning strategies that actively engage students in the process, focus on the learning process instead of the outcome, and assessment techniques that give students intellectual challenges instead of those concerned with memory or memorization.

Introducing non-routine problems with conflicting information to students will have a space to develop their critical thinking skills [33]. Other ways are to involve them in formulating hypotheses, independent learning, presentation, evaluating different points of view, determining the right reasons, and drawing conclusions [46], [47]. Teachers are required to provide metacognitive practices when students experience learning difficulties. The importance of metacognitive practices in the learning environment as an effective means to develop critical thinking skills [48], [49]. Students must be accustomed to working on worksheets in group discussions with guidance from the teacher [50], [51]. By practicing these methods, it is expected that students will avoid making any errors and automatically improve their critical thinking skills in mathematics.

CONCLUSIONS AND RECOMMENDATION

A study on the critical thinking skills of students in Grade 8 has been carried out. The findings show that the current skills level can be categorized as “low”. This is evident from analyzing the three sub-skills of critical thinking: the ability to analyze, evaluate, and draw a conclusion. Furthermore, among these three indicators, it can be concluded that the ability to conclude is the lowest.

The findings of this study are expected to provide an overview to teachers and researchers about the critical thinking skills of students in Grade 8 in solving a system of linear equations with two variables (SLETV). It is recommended for teachers and researchers to design and construct learning and instructional models to improve students' critical thinking skills, especially in analyzing, evaluating, and drawing conclusions.

REFERENCES


