



Students' Critical Thinking Profile in Solving Problems Proving Trigonometric Identities in View of Gender Differences

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Abstract

This study aims to describe the critical thinking profile of SMAN 5 Model Palu grade XI students in solving trigonometric identity proof problems regarding gender differences. The subjects in this study were male students or S1 and female students or S2 of grade XI Mipa 4 SMAN 5 Model Palu who have high mathematical abilities. Data were collected through tests and interviews. The results of this study indicate that S1 and S2 in solving trigonometric identity proofs have the correct answers and are accompanied by supporting reasons. However, there are some differences in thinking activities between the two, namely S1 who is a male student with high ability has a shorter answer than the solution of S2 who is a female student with high ability, S1 uses a logical way of thinking compared to S2 who tends to be proficient with his verbal abilities. This shows that S1 and S2 even though they have equal mathematical abilities, there are some differences between the two in solving trigonometric identity proofs whose data are obtained from written assignment answers and interview results.

INTRODUCTION

Thinking is an activity in everyday life that humans do when they are going to solve a problem. Thinking is a series of mental activities consisting of receiving, processing, storing, and recalling information to solve problems [1-4]. In line with the opinion above, Siswono [5] stated that thinking is a mental activity experienced by someone when faced with a problem or situation that must be solved.

In mathematics learning, a problem is a challenging issue or question that cannot be solved with routine procedures that are usually done or already known. This is to the opinion of Mahardhikawati et al. [3] a question is called a problem for a student if: (1) the student can understand the question faced, but the question must be a challenge for him to answer, and (2) the question cannot be answered with routine procedures that the student already knows. Along with the complexity of problems in everyday life, critical thinking skills become a separate requirement.

According to Siregar & Yolanda [6], critical thinking skills are the ability to evaluate evidence, assumptions, logic, and language from statements or information that is read and heard. Critical thinking skills emphasize meaningful learning because students must learn independently to build their own knowledge.

Machmud [7] stated that in proving a mathematical truth, students will carry out thinking activities, one of which is critical thinking. Therefore, the process of completing mathematical proofs is one way that can be taken to develop students' critical thinking skills. However, proof in mathematical material is not easy, so students' critical thinking skills can still be said to be low.

This can be seen in the Trends in International Mathematics and Science (TIMSS) study in 2018, which showed that Indonesia was ranked 7th from the bottom, 73 out of 79 other countries that participated in TIMSS, with an average estimate of 397. The Programme for International Student Assessment (PISA) score in Indonesia also did not experience substantial development, which was always below 400. This shows that Indonesia needs to repackage its educational context, especially in mathematics lessons regarding mathematical problem-solving skills [8-13].

In addition, many students complain when solving problems related to mathematical proof. For example, what researchers obtained when observing SMAN 5 Model Palu during the learning process related to the Trigonometric Identity proof material, researchers gave students a test question $3 \cos^2 a - 2 = 1 - 3 \sin^2 a$. At the same time, students were working on the test questions. There were differences between female and male students in responding to solving the proof questions; female students could choose logical, relevant, and accurate arguments and behave systematically and in an orderly manner with parts of the whole problem.

1. $3 \cos^2 a - 2 = 1 - 3 \sin^2 a$
2. $3(1 - \sin^2 a) - 2 = 1 - 3 \sin^2 a$
3. $3 - 3 \sin^2 a - 2 = 1 - 3 \sin^2 a$
4. $1 - 3 \sin^2 a = 1 - 3 \sin^2 a \rightarrow \text{terbukti sama}$

Figure 1. Female Students' Answers in Solving Trigonometric Identity Proof Problems

The results of the male students' test work showed that they were able to choose logical, relevant, and accurate arguments, but they were not systematic in the parts of the overall test questions given.

Dimulai dengan sisi kiri persamaan: $3 \cos^2 a - 2$
 Dengan menggunakan identitas $\cos^2 a + \sin^2 a = 1$, kita dapat menulis ulang $\cos^2 a$ sebagai $1 - \sin^2 a$
 distribusi 3 ke dalam kurung: $3 - 3 \sin^2 a - 2$
 menggabungkan suku-suku yang sama: $1 - 3 \sin^2 a$
 Dengan demikian, sisi kiri persamaan disederhanakan menjadi $1 - 3 \sin^2 a$, yang sama dengan sisi kanan. Dengan demikian, identitas $3 \cos^2 a - 2 = 1 - 3 \sin^2 a$ terbukti.

Figure 2. Male Students' Answers in Solving Trigonometric Identity Proof Problems

The low ability of students in mathematical proof is a problem that must be solved. The first step must be to examine the factors that influence the ability to do mathematical proof [13-18].

METHODS

This research falls into the category of qualitative research because it naturally aims to examine in-depth the profile of students' critical thinking skills on Trigonometric Identity material, viewed from gender differences. This qualitative approach enables researchers to explore rich and detailed data regarding how students of different genders process information, understand concepts and apply logic in solving trigonometric problems. This research focuses on describing phenomena in detail based on the data collected from the research subjects without manipulation or experimental intervention.

A descriptive approach provides a clear picture of students' critical thinking profiles. Through this approach, researchers aim to present data or information as it is, based on the situations found in the field. Data is collected through various methods, such as interviews, observations, or document analysis, to explore how students demonstrate critical thinking, the emerging patterns, and the characteristics differences based on gender. This approach is highly relevant as it allows researchers to understand the context and complexity of the interaction between gender and critical thinking skills in mathematics learning.

The results of this study are expected to provide deeper insights into the dynamics of students' critical thinking in the context of trigonometry learning, particularly concerning gender influences. With this understanding, educators can design more inclusive and effective teaching strategies that support the development of student's critical thinking skills regardless of gender differences. Furthermore, this research contributes to developing educational theories that consider psychological and sociological aspects of the teaching and learning process.

RESULTS AND DISCUSSION

This research was conducted at SMAN 5 Model Palu located on Jl. RE. Martadinata, Palu, Central Sulawesi. The time of this research was Wednesday, January 17, 2024, and Friday, February 02, 2024. Data collection was conducted face-to-face between the researcher and the research subject. The researcher gave a trigonometric identity proof test, which aimed to group students based on their mathematical abilities, in addition to determining the critical thinking process of students in completing the trigonometric identity proof. The test consisted of two questions that students worked on for 45 minutes. In the second activity, the researcher conducted interviews with selected subjects. The interviews conducted in this study were task-based. Interviews were conducted to learn more about the subject's critical thinking process in completing the trigonometric identity proof. The critical thinking process at the interpretation stage can be seen through the subject's written answers and supplemented with interviews with the subject. The critical thinking process at the analysis stage can be seen through written answers and interviews with the subject. The critical thinking process at the evaluation stage can be seen through interview answers and written answers. The critical thinking process at the inference stage can be seen through the subject's written answers and supplemented by interviews.

1. The critical thinking ability of male students in solving trigonometric identity proof problems in the interpretation indicator is shown by students being able to write/mention what is known and asked in the problem, namely writing what is known $\sin a + \cos a \cot a = \csc a$

and $\frac{1+\sec a}{\tan a + \sin a} = \csc a$, and writing what is asked in the problem, namely proving trigonometric identity, this is in line with Susilowati [24] who argues that in understanding the problem, male students collect written facts in the problem by mentioning things that are known and asked. Then, the analysis indicator shows that students can write each step used in solving the problem. S1 writes the steps to solve the problem by first changing the equation that can be changed using the trigonometric identity formula. In question number one, S1 changes it $\cot a$ $\frac{\cos a}{\sin a}$, so that the initial equation form $\sin a + \cos a \cot a = \csc a$ after S1 changes it $\cot a$, the equation form changes to $\sin a + \cos a \cdot \frac{\cos a}{\sin a} = \csc a$. For the next step, S1 equates the denominators by multiplying so that $\sin a + \cos a \cdot \frac{\cos a}{\sin a}$ they get the equation form $\frac{\sin^2 a + \cos^2 a}{\sin a}$ then in the equation form that S1 has obtained, the equation $\sin^2 a + \cos^2 a$ can be changed using the trigonometric identity formula, namely $\sin^2 a + \cos^2 a = 1$. After being changed, the equation form becomes $\frac{1}{\sin a}$, in the trigonometric identity formula $\frac{1}{\sin a} = \csc a$ so that it is proven that $\sin a + \cos a \cot a = \csc a$. Then in number two S1 changes $\sec a$ to $\frac{1}{\cos a}$ and $\tan a$ becomes $\frac{\sin a}{\cos a}$ after being changed from the initial equation form, namely $\frac{1+\sec a}{\tan a + \sin a} = \csc a$ to $\frac{1+\frac{1}{\cos a}}{\frac{\sin a}{\cos a} + \sin a}$ equate the denominator S1 multiplies $\frac{1+\frac{1}{\cos a}}{\frac{\sin a}{\cos a} + \sin a}$ by the equation $\frac{\cos a}{\cos a}$ after multiplying S1 gets the equation $\frac{\cos a + 1}{\sin a + \sin a \cos a}$, then S1 changes $\sin a \cos a$ to $1 + \cos a$ so that the equation form becomes $\frac{1 + \cos a}{\sin a + (1 + \cos a)}$ in this equation there is one that can be operated, namely the equation $1 + \cos a$, after being operated the equation form can be written $\frac{1}{\sin a}$ in the identity formula $\frac{1}{\sin a} = \csc a$ then it is proven that $\frac{1+\sec a}{\tan a + \sin a} = \csc a$.

Furthermore, the evaluation indicator is shown by students being able to use the right strategy in solving problems, namely finding the trigonometric identity formula to change the equation form of the problem. After that, it is operated to make it easier to get the final result. The inference indicator is shown by students being able to draw and write conclusions based on each step in solving the problem. Thus, this is in line with research conducted by Wiranta [19], which states that male students can logically draw conclusions about a problem.

The critical thinking ability of female students in solving trigonometric identity proof problems on the interpretation indicator is shown by students being able to write/mention what is known and asked in the problem, namely mentioning what is known $\tan \theta - \cot \theta$ and $\tan x \sin x + \cos x$ also mentioning what is asked in the problem, namely $\tan \theta - \cot \theta$ whether if operated will produce $\frac{1-2\cos^2 \theta}{\sin \theta \cos \theta}$ and $\tan x \sin x + \cos x$ whether if operated will produce $\sec x$. Then, on the analysis indicator, it is shown that students can write each step used in solving the problem by first changing the equation that can be changed using the trigonometric identity formula as in question number one, S2 changes $\cot a$ to $\frac{\cos a}{\sin a}$ after being changed the initial equation $\sin a + \cos a \cdot \cot a = \csc a$ becomes $\sin a + \cos a \cdot \frac{\cos a}{\sin a} = \csc a$, then to equalize the

denominator S2 multiplies $\sin a + \cos a \cdot \frac{\cos a}{\sin a}$ so that the equation becomes $\frac{\sin^2 a + \cos^2 a}{\sin a}$ this equation there is one that can be changed using the trigonometric identity, namely $\sin^2 a + \cos^2 a = 1$. The equation obtained becomes $\frac{1}{\sin a}$, in the trigonometric identity $\frac{1}{\sin a} = \csc a$ so that it is proven $\sin a + \cos a \cdot \cot a = \csc a$. In question number two, S2 changes the equation $\sec a$ to $\frac{1}{\cos a}$ and $\tan a$ to $\frac{\sin a}{\cos a}$ so that from the initial equation $\frac{1 + \sec a}{\tan a + \sin a} = \csc a$ it becomes $\frac{1 + \frac{1}{\cos a}}{\frac{\sin a}{\cos a} + \sin a} = \csc a$. The next step, S2, $\frac{1 + \frac{1}{\cos a}}{\frac{\sin a}{\cos a} + \sin a}$ by $\frac{\cos a}{\cos a}$ equating the denominators. After multiplying the equations, the form of the equation is obtained $\frac{\cos a + 1}{\sin a + \sin a \cos a}$ in the equation. Some can be changed, namely $\sin a \cos a = 1 + \cos a$, so that the form of the equation becomes $\frac{1 + \cos a}{\sin a + (1 + \cos a)}$ then there is something that can be changed, namely $1 + \cos a$, so that the equation becomes $\frac{1}{\sin a}$ in the trigonometric identity formula $\frac{1}{\sin a} = \csc a$ then it is proven that $\sin a + \cos a \cdot \cot a = \csc a$. The evaluation indicator is shown by students being able to use the right strategy in solving problems, namely finding the trigonometric identity formula that matches the form of the equation to be changed. After that, it is operated to make it easier to get the final result. Then, the inference indicator is shown by students' ability to draw conclusions based on each step in solving the problem. Thus, this aligns with research conducted by Palobo and Nur [20], which states that female students use problem-solving steps that have been planned correctly and obtain accurate answers.

CONCLUSION

Based on the research results and discussions presented in the previous chapter, the conclusions regarding this research are as follows.

1. Critical Thinking Profile of Male Students with High Mathematical Ability

Based on the results of the answers and interviews of high-ability male students, namely S1, it was concluded that S1 met all critical thinking indicators, starting from the interpretation indicator, namely S1 wrote down the information known and asked in the question correctly, in the analysis indicator S1 could write down each step used in solving the problem correctly, in the evaluation indicator S1 used the right strategy in solving the problem and was complete and correct in doing the calculations, in the inference indicator S1 made conclusions correctly. This shows that S1 met all critical thinking indicators.

2. Critical Thinking Profile of Female Students with High Mathematical Ability

Based on the results of the answers and interviews of high-ability female students, namely S2, it was concluded that S2 met all critical thinking indicators, starting from the interpretation indicator, namely S2 stating what is known and asked in the question correctly, in the analysis indicator S2 can write down each step used in solving the problem correctly, in the evaluation indicator S2 uses the right strategy in solving problems and is complete and correct in calculating,

in the inference indicator S2 makes conclusions correctly. This shows that S2 meets all critical thinking indicators.

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