Volume 4, Nomor 1, Juni 2020
Numerical: Jurnal Matematika dan Pendidikan Matematika
http://journal.iaimnumetrolampung.ac.id/index.php/numerical
DOI:https://doi.org/10.25217/numerical.v4i1.702

Eigen Problem Over Max-Plus Algebra on Determination of the T3 Brand Shuttlecock Production Schedule

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Article Info
Article History
Received: 06-01-2020
Revised: 07-06-2020
Accepted: 08-06-2020
Keywords:

Max-Plus Algebra;
Production System;
Scheduling

Abstract

The production process is included in the Discrete Event System (DES). The DES independent variable generally depends on the event, so an event is influenced by the previous event. Max-plus algebra can be applied in the DES problem to change the system of nonlinear equations obtained into linear equations. Max-plus algebra is a set of real numbers $\mathbb{R}$ combined with $\varepsilon = -\infty$ equipped with operations max $\oplus$ and plus $\otimes$ or can be denoted $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ with $\mathbb{R}_\varepsilon = \mathbb{R} \cup \{\varepsilon\}$. An effective and efficient production process needs to pay attention scheduling steps well. The purpose of this research is to determine the Shuttlecock T3 production schedule using eigenvalue and eigenvector in max-plus algebra. The research method in this research is study of literature and observation. Literature study is carried out by studying references about max-plus algebra, especially material related to scheduling problems, while observation are carried out in the process of taking data of the Shuttlecock T3 production process in Surakarta. The linear equation system that is formed based on the results of the observation is then presented in the form $x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1)$ and $y(k) = C \otimes x(k)$. The periodic time and initial system production time are determined from the eigenvalue and eigenvector matrix $\bar{A}$ where $\bar{A} = A \oplus B \otimes C$. The results of the research showed that the production system run periodically every 249 minutes, then the best time for each processing unit to start working can be determined, as well as the Shuttlecock T3 production schedule according to the working hours more effective and efficient can be determined too.

PRELIMINARY

Every company cannot avoid competition with other companies. Other companies in this case are known as competitors, namely companies that offer almost similar goods or services [4]. Competition between companies not only gives a negative influence on the perpetrators, but also gives a positive influence in the form of the development of the industrial world as it is today. A company is trying to improve product quality according to consumer demand and carry out the production process effectively and efficiently in order to win trade competition. An effective and efficient production process needs to pay attention to the steps of planning, monitoring and scheduling properly. The production process forms a complex system, this system is included in the Discrete Event System (DES).

According to Silva [10], DES is a subject of the discipline of systems theory and control that includes man-made systems with a finite amount of resources. Existing resources are used by several users to get the best results. The DES independent variable generally depends on the event, not on time, so that an event is influenced by a previous event, not by the current time. Nowak [7] also explained that DES can be used to solve problems such as determining production schedules. Max-plus algebra can be applied to the problem to change the system of nonlinear equations obtained from the problem model into a system of linear equations so that it

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is easily solved. The concept of Eigen values and Eigen vectors, or what is referred to as Eigen value over max-plus algebra, can also be used as a reference in determining the schedule of a production system.

One product that develops in Surakarta is shuttlecock. The shuttlecock industry is a superior product in the Serengan area, which is one of the districts in Surakarta. Shuttlecock craftsmen in the area reach hundreds of people. The T3 brand sshuttlecock, which was established in 1988, is one of the industries which has enlivened the market in the area. The longer the demand for shuttlecocks in the area around Surakarta increased. This increase in demand needs to be supported by improving production system management. Determination of an effective and efficient production schedule is one of the requirements for good management so that this problem then becomes the basic idea of the research and the main objective of this research.

Research Kamceva [3] explained that max-plus algebra could complete the DES model. Then Mursyidah [6] and Sulistyaningsih [12] has also explained that max-plus algebra can be used to solve scheduling problems. Furthermore, in 2016 Sari and Pradanti [9] examined the application of max-plus algebra in a simple leather bag production system, in the same year Muntohar [5] researched the application of max-plus algebra in scheduling the Solopos general daily production system at PT. Solo Grafika Utama.

Some of these studies still use a small number of processors and the type of production system used is still serial and parallel with one input. In contrast to previous studies, in this study more processors were used, namely 20 units and the research object was converted into a production process on a T3 brand shuttlecock that uses an assembly type production system with two inputs in it so that it better illustrates the real problem in the field. Based on the background of the problem, the formulation of the problem is determined by constructing a matrix from the flowchart, solving Eigen problems, determining the production schedule, and knowing the effect of applying the production schedule.

METHOD

The research method used in this research is a field study by conducting the data collection process in the form of the T3 brand shuttlecock production flow in Surakarta and the work time of each processor. The steps that will be carried out in this research, namely starting with the T3 brand shuttlecock production process data flow and the working time of each processor, then making a production process flowchart from the data obtained and building a matrix of max-plus algebra. The matrix formed is then determined by the Eigen value and eigenvector vector as a reference in determining an effective, efficient and periodically repeatable shuttlecock production schedule.

RESULT AND DISCUSSION

Based on the results of the field study, the flow data of the T3 brand shuttlecock production process and the working time of each processor are shown in Table 1. The T3 brand shuttlecock production process consists of 20 processing units namely $P_1, P_2, P_3, \dots, P_{20}$. The processing time needed for processors $P_1, P_2, P_3, \dots, P_{20}$ are $d_1, d_2, d_3, \dots, d_{20}$ in minutes. There are two inputs in it, namely the input of raw materials in the form of feathers and the second input is

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the basic material for making hubcaps. The processing time of each processor is obtained by taking the average value of four times manually sampled data in the field.

Table 1. Production Flow of T3 Brand Shuttlecocks and Time of Each Process

P_i	Process	d_i (minute)
P_1	Feather Printing	26
P_2	Cutting fur	30
P_3	Fur Wash	10
P_4	Drying Fur	14
P_5	Sorting Fur	6
P_6	Hair Straightening	32
P_7	Make Hubcaps	8
P_8	Feathering on Hubcaps	36
P_9	Gluing Fur with Hubcaps	6
P_{10}	Tie the Fur	34
P_{11}	AdjustmentShuttlecock	14
P_{12}	StampingShuttlecock	2
P_{13}	Thread Gluing	6
P_{14}	Drying	10
P_{15}	Quality Control	12
P_{16}	Sorting Based on Quality	6
P_{17}	Labeling	1
P_{18}	Insert Shuttlecock in Slope	2
P_{19}	Packing	1
P_{20}	Sealing	1

Then the production flow chart based on the results of the field study is shown in Figure 1. This flow chart can facilitate the construction of a non linear equation system model that describes the T3 brand shuttlecock production system [13].

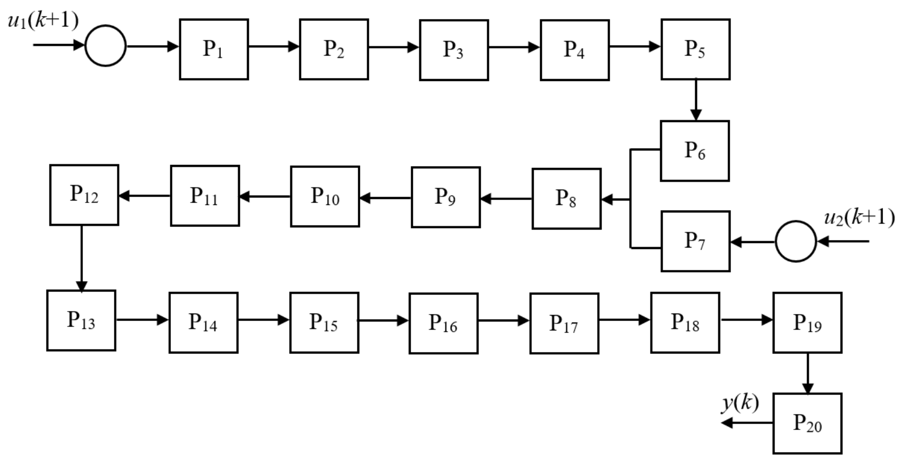

Figure 1. T3 Shuttlecock Production Flowchart

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Print ISSN: 2580-3573, Online ISSN: 2580-2437

The form of the development of this research appears in Figure 1 that the data represented in chart form is the type of assembly shown in the P_6 and P_7 branches. Previously Muntohar [5] in his research only used serial types so that no branching was found in the process. Furthermore, there are two raw material inputs, namely at $u_1(k+1)$ and $u_1(k+1)$ which were also not found in previous studies. Defined the T3 brand shuttlecock production system as follows

1. $u(k+1)$ is the time when raw material enters the system for process $(k+1)$ with $k = 1, 2, 3, \ldots$,
2. $x_i(k)$ is the time when the unit i starts working for the process k with $i = 1, 2, 3, \ldots, 20$, and
3. $y(k)$ is the time product leaves the system for process k.

In some previous studies the maximum amount of data used was only 12 units so that a square matrix with order 12 would be formed, whereas in this research data 20 data processing units would be used so that the matrix number 20 would be formed. Determination of the time P_1 started working on process to $(k+1)$ is explained as follows. If the raw material is entered into the system for process $(k+1)$, then the raw material is available as input P_1 at time $u_1(k+$ *1*. However, $P1$ can only start working to process the raw material if $P1$ has completed the previous process, the k-th process. Because the processing time needed for P_1 is $d_1 = 26$ minutes, the semi-finished product from P_1 leaves P_1 when $x_1(k) + 26$. The time obtained when P_1 starts working for process to $(k+1)$ is

$$\boldsymbol{x_1(k+1) = \max\{x_1(k) + 26, u_1(k+1)\} = 26 \otimes x_1(k) \oplus u_1(k+1)}.$$

And so on in the same way for $P_2, P_3, P_4, \ldots, P_{20}$ so that 20 linear equations are obtained for max-plus algebra. The system of equations obtained is then modified in matrix operations on algebra max plus as described Gyamerah [2], it becomes a form

$$\boldsymbol{x}(k+1) = A \otimes \boldsymbol{x}(k) \oplus B \otimes \boldsymbol{u}(k+1)$$
$$y(k) = C \otimes \boldsymbol{x}(k).$$

It is assumed that the time when raw materials enter the system together with the time when the product leaves the system, or can be written $u(k+1) = y(k)$ so that obtained $\bar{A} = [A \oplus B \otimes C]$.. Then the production process schedule can be formed by determining the periodic time of the production process from the Eigen values of matrix $\bar{A}$ and the starting time of each processor starts working is determined from the eigenvector matrix $\bar{A}$ [8]. Using the algorithm described by Subiono [11] and Awallia [1] and using the help of Scilab software, the results as shown in Figure 2. These results require more iterations compared to previous studies, because the amount of data is greater and the type of matrix may be reducible so that the existence and singularity of Eigen values need to be checked first.

$$\lambda = \frac{c}{p-q} = \frac{249}{3-2} = 249$$

$$v = \begin{pmatrix} 26 & 52 & 82 & 92 & 106 & 112 & 112 & 144 & 180 & 186 & 220 & 234 & 236 & 242 & 252 & 264 & 270 & 271 & 273 & 274 \end{pmatrix}^T$$

Figure 2. EigenvalueandEigenvector of Matrix$\bar{A}$

The initial time the system starts working is selected with an eigenvector that has a minimum non-negative component so that the finished process and available materials can be processed immediately. An eigenvector with a minimum non-negative component is obtained when the smallest Eigen vector component is converted to zero [7]. The eigenvector v'which has the minimum non negative component is obtained by taking $\alpha = -26$ so that v' is obtained as in Figure 3.

$$v' = \begin{pmatrix} 0 & 26 & 56 & 66 & 80 & 86 & 86 & 118 & 154 & 160 & 194 & 208 & 210 & 216 & 226 & 238 & 244 & 245 & 247 & 248 \end{pmatrix}^T$$

Figure 3. Corresponding Eigen Vector

So for each processing unit will work periodically with a period $\lambda(\bar{A}) = 249$ minutes and the processing units $P_2, P_3, P_4, \dots, P_{20}$ successively work for the first process at minute $0,26,56, \dots ,248$. The full production schedule is shown in Table 2.

Table 2. Production Schedule in Minutes

Process	Phase	
	1	2
P_1	0	249
P_2	26	275
P_3	56	305
P_4	66	315
P_5	80	329
P_6	86	335
P_7	86	335
P_8	118	367
P_9	154	403
P_{10}	160	409
P_{11}	194	443
P_{12}	208	457
P_{13}	210	459
P_{14}	216	465
P_{15}	226	475
P_{16}	238	487
P_{17}	244	493
P_{18}	245	494
P_{19}	247	496
P_{20}	247	497

From Table 2, it can be converted into working hours to be more representative in accordance with the actual situation on the ground. Based on the results of the field study, the T3 brand shuttlecock starts at 07.30 WIB so that the 0-th minute in Table 2 can be represented that

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Print ISSN: 2580-3573, Online ISSN: 2580-2437

the first processor (P_1) starts working at 07.30 WIB and so on for the other processors. The production schedule after referring to working hours is shown in Table 3.

Table 3. Production Schedule of T3 Brand Shuttlecock

Process	Time to Start Production (WIB)	
	Phase1	Phase2
P_1	07.30	11.39
P_2	07.56	12.05
P_3	08.26	12.35
P_4	08.36	12.45
P_5	08.50	12.59
P_6	08.56	13.05
P_7	08.56	13.05
P_8	09.28	13.37
P_9	10.04	14.13
P_{10}	10.10	14.19
P_{11}	10.44	14.53
P_{12}	10.58	15.07
P_{13}	11.00	15.09
P_{14}	11.06	15.15
P_{15}	11.16	15.25
P_{16}	11.28	15.37
P_{17}	11.34	15.43
P_{18}	11.35	15.44
P_{19}	11.37	15.46
P_{20}	11.38	15.47

Based on Table 3, it is known that in one working day two production processes can be carried out. The first starts at 07.30 WIB and then the second production starts at 11.39 WIB. Using the production schedule in Table 3 allows the T3 brand shuttlecock business owner to know when a good start time to start each processor works and the production process can run periodically.

CONCLUSIONS AND RECOMMENDATION

This research produces the T3 brand shuttlecock production schedule. The production schedule makes the production process more effective and efficient because in the production process, the idle processor will immediately start the next process. In terms of management it also makes it easy for T3 brand shuttlecock business owners to know when a good start time to start each processor works and the production process can run periodically. However, this research is still limited to the assumption that the time when raw materials enter the system together with the time when products leave the system. Therefore, further research is needed so that it can refine these assumptions to better suit real-world events.

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